LEARNING THE FUNCTION CONCEPT IN AN AUGMENTED REALITY-RICH ENVIRONMENT

Yael Levy[1], Otman Jaber[1,2], Osama Swidan[1] and Florian Schacht[3]

[1]Ben-Gurion University of the Negev, Israel; [2]Al-Qasemi Academic College of Education, Israel, zeta275@yahoo.com; [3]University of Duisburg-Essen, Germany

Even though the function concept is central in mathematics and is included in many curricula worldwide, many students find it difficult to understand. In this study, we adopt the function concept as a covariation between two variables. This study aims at exploring students' understanding of covariation while learning in an augmented reality (AR)-rich environment. Four groups of three 16-year-old students participated in this study. Each group carried out the Hooke's Law experiment using AR headsets. The students' interactions were video-recorded, and semiotic lenses were used to analyze their covariational reasoning. Findings show that the AR-rich environment promoted the students' covariational reasoning with mostly elementary levels, but also with some indications of high levels.

Keywords: augmented reality, covariational reasoning, multimodality

INTRODUCTION

The function concept is central in mathematics and is included in many curricula worldwide. However, many students find it difficult to understand and graduate from high school with a lack of knowledge of this concept (Akkus, Hand and Seymour, 2008). Several studies have been conducted on teaching the function concept, some using digital technologies that include computers and simulations. These studies found that the use of dynamic technology tools fosters students' understanding of the function concept (e.g., Hoffkamp, 2011). Even though several attempts have been made in the recent years to integrate AR technology into science and mathematics education (e.g., Yen, Tsai and Wua, 2013), less is known about the AR affordances to foster pre-calculus concepts. In this paper, we aim at shedding light on the role of augmented reality technology in fostering students' understanding of the function concept. For this reason, we designed an innovative AR tool and explored its effectiveness in fostering covariational reasoning as an indication of understanding the function concept.

The study reported here is innovative for two reasons. (1) It proposes a new prototype for employing augmented reality in an educational setting using a special headset, presenting a dynamic object in a real environment with virtual representations. In contrast to the design presented in this paper, typical ways of using this technology involve augmenting static objects rather than dynamic ones (for example, a 3D view of a cell when observing a cell picture on a biology book page). (2) It explores covariational reasoning as mathematical representations juxtaposing a dynamic real-world object, showing that it is innovative in mathematics education.

THEORETICAL FRAMEWORK

Covariation reasoning: In this study, we addressed the function concept as a dynamic process of covariation. Thompson and Carlson (2017) described understanding covariation as holding a sustained image in the mind of two quantities' values (magnitudes) that change simultaneously. They discussed understanding the function concept by describing the meanings and thinking styles that could be attributed to someone who understands the essence of the function. In their study, they described five levels of covariation: (1) pre-coordination of values; (2) gross coordination of values; (3) coordination of values; (4) chunky continuous covariation; and (5) smooth continuous covariation. In the first level, the student can predict the change of each variable value separately but cannot create pairs of values. In the second level, the student perceives a loose link between the overall changes in the two quantities' values, such as "this quantity increases as the other decreases." In the third level, the student can match values of one variable (x) to values of another one (y), creating a discrete set of pairs (x, y). In the fourth level, the student may perceive that the changes of two variables occur simultaneously, and that they vary in piecewise continuous covariation. In the fifth level, the student can perceive that an increase or decrease in the value of one variable occurs simultaneously with changes in the value of the other variable and see that both variables change smoothly and continuously. In this paper, we observe students' actions regarding the function concept through the theoretical lens of the Action, Production and Communication (APC) space.

Multimodality and the APC space: The term "multimodality" refers to the importance and mutual coexistence of a variety of cognitive, material and perceptual resources or perceptions in the mathematics learning processes, and in general, in the creation of mathematical meanings. Radford, Edwards and Arzarello (2009) argued that "these resources or means include verbal and written symbolic communication, as well as drawing, gestures, manipulation of physical and electronic devices, and various types of physical movements" (ibid, pp. 91-92).

Earlier studies have shown that gestures play an important role in mathematics education when students solve problems and explain mathematical concepts (e.g., Edwards, 2009). Gestures are just part of a whole arsenal of students' resources available to bridge their experiences with daily life phenomena and formal mathematics (Arzarello & Sabena, 2014).

The data analysis in this study was done according to the APC space perspective (Arzarello and Sabena, 2014). This method considers multimodal resources and analyzes learning processes to understand conceptual knowledge. The APC space model consists of three main components: the body, the physical world and the environment. These components are vital in mathematical activities in the classroom's social context. Hence, it is important to examine them in analyses of learning processes. The three crucial APC components (action, production and communication) of students' learning processes in mathematics highlight their active role in the learning process (Arzarello and Sabena, 2014). According to this perspective, a suitable mathematical learning environment for students must include three specific activities: action and interaction (e.g., with classmates, the teacher, tools or the students themselves); production (e.g., answering or asking questions, conjecturing); and communication (e.g., when a solution is delivered to a teacher or classmate, verbally or in writing, using appropriate representations). The APC space considers the environment in which students interact – including the tools they use – as being crucial for learning, thinking and the inquiry process. During our analysis, we addressed all multimodal resources that the students considered in the learning process: speech, body gestures, drawings and interactions with the physical model.

Augmented reality: AR is an innovative technology that incorporates a wide variety of techniques for presenting computerized materials (such as text, images and video) about the real world as seen in the normal state (Kaufmann et al., 2005). AR combines several layers of virtual objects that are augmented over physical objects in the real world, creating a unique reality in which virtual objects and the real environment coexist. AR technology has many advantages. It contributes greatly to student learning because it integrates the benefits of physical and virtual learning experiences (Bujak et al., 2013). It can help students learn challenging scientific content thanks to its ability to present information and details visually that are not naturally visible. In addition, it allows students to experience interactive 3D simulations, leading to deeper insights about phenomena that might be difficult to understand; it simplifies objects' visual appearance and helps students think about their symbolic representations; and it enables students to observe virtual objects in a perspective they choose while still being able to see other students (Bujak et al., 2013). Two research questions guided this study: (1) What levels of covariational reasoning arise among students as they learn in an AR-rich environment? (2) How does the use of an AR-rich environment facilitate covariational growth?

METHOD

Research context and participants: This study is based on the qualitative research method. The research experiments were conducted with four groups, each containing three 10th- and 11th-graders studying in two high schools in southern Israel. The participants study in their schools advance mathematical topics (geometry, algebra, and calculus). They had studied linear functions in 8th grade and quadratic functions in 9th grade. The meetings were held in a scientific laboratory at Ben-Gurion University of the Negev. Each session lasted for 90-120 minutes. Each group carried out a physical experiment, the Hooke's Law experiment,

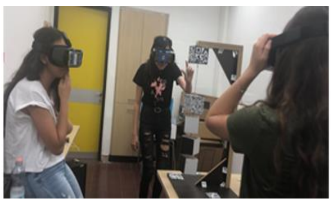

Figure 1. Hooke's law experiment

which examines the relationship between mass and elongation of a spring (Figs. 1, 2). At the beginning of the experiment, students received an explanation about each part of the experiment, as well as about using the technology. Each group worked on task sheets (see link) corresponding to both physical experiments.

Figure 2. Virtual data as seen through the AR headset

Data collection and analysis: All learning activities were video-recorded. Students' interactions, gestures and materials (written notes, files) were collected. Thus, a solid set of data was obtained. Data were analyzed using the deductive approach (Patton, 2002). In the first phase, all videos were observed to get a general impression about the process. The second analysis phase refers to the levels of covariational reasoning (Thompson and Carlson, 2017). The third phase involved transcription of the observations. Students' statements and interactions – peer-to-peer interactions or interactions with the model – were recorded and documented. The fourth phase refers to accurate encoding of transcripts and the search for statements and instances of our data categories, which included scans of transcripts to identify expressions that might indicate any level of covariational reasoning. These expressions were categorized into appropriate levels of covariational reasoning.

FINDINGS

Covariation level 2 - the evolution of quantities: This example illustrates how students were engaged in the second level of covariational reasoning. It happened when they coordinated the weight of cubes and the shape of the graph. Tal, Hila and Maya wore the AR headset and observed the spring elongation. After identifying the virtual object that displayed the length of the spring (blue line in Fig.2), they added cubes one by one while looking through the headset so that they could observe the variation in spring length. Twenty minutes after beginning the experiment, Hila noticed for the first time that the graph began to appear on the screen. She went on to say, "It seems to me that if we add more cubes, the function will continue." Later on, the girls changed the spring with a new one and again added cubes. The changes were observed simultaneously through the AR headset. Here, Maya also saw the graph and described its direction through body gestures (a sloped increasing hand movement) while stating "it is from zero to 10" after continuing to add cubes, Hila reported (excitedly) that she saw the graph "from zero – absolutely increasing... really inclines to the side."

Afterwards, the researcher asked them to share their insights.

Hila: The more weight we add, the greater the graph function.
Maya and Hila: It inclined to the right. (gesture with their hands to the right (Fig. 3a).
Researcher: What do you mean?

Hila: If the graph started out like this, the more I add, the more it inclines to the right (Fig. 3b).

Maya: Increasing.

Hila: Yes, increasing, pulling more up like this.

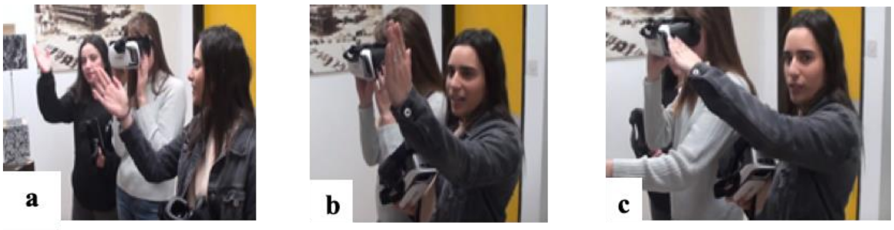

Figure 3. (a) Hila and Maya gesture the inclination to the right"; (b) Hila gestures "if the graph started out like this"; (c) Hila gestures the inclination to the right."

Hila explains her insights and says: "as we add weight" and refers to hanging more cubes on the spring, "The graph inclines more to the right." Maya agrees with Hila's insight. Tal, the third student, does not take part in the discussion and continues to observe the model through the AR headset. It is interesting to note that Hila uses non-mathematical terms to describe the graph as "inclining to the right." The gestures in Fig. 3b and 3c, suggests that the students express the graph's rotation without using the word "slope." Maya immediately adds that the graph is "increasing" and adds a mathematical element here to the description Hila had provided.

In this example, Hila and Maya describe a relationship between two objects – cube weight and graph shape. During the experiment, they hung cubes one by one while observing the graph generated by the AR. As a result of their actions, they concluded that "as we add weight, the graph inclines to the right." This type of covariation could refer to the second level, because students see general changes in the values of two quantities, but do not coordinate specific pairs of values. As we will argue in the Discussion, this phenomenon can be seen as one of the greatest potentials of using AR: in this example, the students deal with a real experiment and add cubes to the spring. They see changes in the graphical visualization simultaneously. They initially use the concept of "weight" – a quantity – to explain the observed phenomenon. Hence, in this example, the quantities the students coordinate (weight and changes over time) emerge while conducting the experiment.

Covariation level 3 - the mathematization process: In this example, we illustrate how Uri, Shilat and Shahar engaged in the third level of covariation they encountered when coordinating between cube weight and spring length. The students performed the experiment using the AR headset and observed that the addition of each cube really increases spring length. "Every time we add 100 grams, we get 5.19 (cm) for the first 100 grams. For the second 200 grams, 6.15. At 300 grams, 6.83 and then 6.00 at 400 grams."

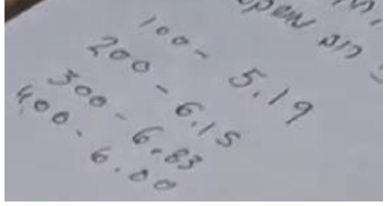

Figure 4. Matching cube weight and spring length

The students performed several actions during the experiment: while Uri and Shilat added cubes and observed the resulting values on the spring through the headset, indicating its length at each moment, Shahar documented the values on the notes page. She prepared a table with the left column representing cube weight and the right column the corresponding spring length value (Fig. 4).

Shahar describes the relationship between two covariation variables: cube weight and spring length. The way she read and wrote the data, it can be concluded that the type of covariation she expresses here is of level three according to Thompson and Carlson (2017). In terms of using AR technology, this example represents an important step in the process of mathematization: the function concept, like all mathematical concepts, is an abstract object. A representation (e.g., graph or table of values) is not a mathematical object itself; it is a representation of an abstract object. Hence, the real phenomenon as well as the mathematical representations are not self-explaining, rather they must be connected conceptually by the students. This example gives insights into the strategies that students use in order to reconstruct a mathematical conceptual meaning behind observed phenomena (real situation and augmented reality experiment).

Covariation level 4 - the multimodal process of meaning-making: In this example, we illustrate how students engaged in the fourth level of covariation they encountered when coordinating between cube weight and spring length. In the Hooke's Law 4 experiment, a few minutes after Xenia concluded that the table and the graph were changing simultaneously, Ronnie explains her insight about the connection between spring length and number of cubes: Ronnie: Here you see this is at 0.7, and here at 0.4, it means that the slope between each other is lower, meaning that at the beginning, the line was more... less… seems steep, more like this... (Fig. 5), and then such a break point, and then it will become more (Fig. 6) ...., and then you understand? It will increase at such a velocity."

Figure 5. Ronnie shows a slight slope with her hand.

Figure 6. Ronnie shows a steep slope with her hand.

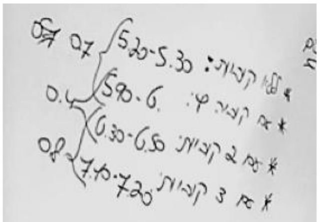

Figure7. Table prepared by the students.

In this example, Ronnie explains her insight to the group members. She finds it difficult to communicate her insight verbally and therefore uses body gestures to convey the message to her classmates. After generating a table connecting between spring length and number of cubes (Fig. 7), Ronnie refers to the difference between the first two rows in the table as being 0.7 and between the second and third rows as being 0.4 ("This is 0.7 here and 0.4 here"). She describes that the graph starts out at a more moderate slope "less steep" (see Fig. 5), and at some point, becomes a section of the graph that is steeper (see Fig. 6). Because Ronnie relies on the table they had prepared together with the differences, we conclude that she links the two variables here – the number of cubes and spring length. This type of coordination may be associated with the fourth level of covariation by Thompson and Carlson (2017) because Ronnie describes the graph as two continuous segment lines separated by a point, which she describes as a "break point" at which the change in slope occurs.

In terms of using AR technology, this example shows an important step in the process of meaning-making: here, the students use a mathematical strategy (determining differences) in order to reconstruct the conceptual (in this case, functional) relationship. Although, in this example, the result leaves questions unanswered (e.g., How to explain the "different differences" 0.4 – 0.7 – 0.8?), it still reveals how students use concepts (like analyzing differences between given quantities) in order to reconstruct mathematical structures.

FINAL REMARKS

The findings show that the participants engaged in the second, third and fourth levels of covariation. We also found that the students mostly engaged in the second and third levels of covariation, and less in the fourth level. This could be attributed to the nature of the Hooke's Law experiment, namely, hanging cubes one by one and observing the spring's elongation the after each addition. Students who focused on the table of values mainly engaged in the third level of covariation, while, students who focused on the length-mass graph mainly engaged in the second or fourth levels of covariation. The second and fourth covariation levels illustrated in examples 1 and 3 differ somehow from the covariational reasoning defined by Thompson and Carlson (2017). In their definition of covariational reasoning, they refer to two values of quantities that vary simultaneously. In the two examples presented in this paper, the students covary quantity (mass) with object (graph) (e.g., the more mass we add the more the graph is inclined). Inspired by Arzarello (2019), to distinguish between the covariational reasoning defined by Thompson and Carlson (2017) we introduce the term "*second-order covariation*," which refers to two objects that covary simultaneously.

The first and third examples help us hypothesize that second-order covariation is complicated just like covariational reasoning. In both the first and third examples, the students resorted to gestures as a semiotic means to thinking with and through them, while in the second example, their thinking was mainly mediated by words and text. We also conjectured that the higher level of second-order covariation is more complex than the lower covariation level. In contrast to the students' gestures in example 1, which were in tune with the students' statements and could be considered as a means of communication to answer the researcher's question, in the third example, gestures exchanged the students' statements, which they were unable to express through their thoughts. Hence, it seems that the students resorted to gestures as a means of thinking (Vygotsky, 1978)

The three examples show that the students continue to interact with the virtual objects displayed by the headset even after removing it. Considering that the students removed the headset after a short time of use, continuing the interaction with the virtual objects suggest that the design of the AR, which juxtaposes real-world objects with virtual objects, was found to be effective for transferring external signs such as the mass-length graph or table of values of mass-length to become instruments for the students (Trouche, 2003).

Other than identifying the covariation levels, the three examples also show results in terms of the *process* of meaning-making. Example 1 shows that the use of AR technology can support the emergence of quantities that the students deal with (Thompson and Carlson, 2017). When the students deal with a real object and a graphical representation, they ascribe a quantity (weight) to the box. In order to make sense of the graph, they introduce a second quantity (length). Such observations must be connected conceptually, and example 2 shows how students organize such a process of mathematization (focused collection of observed quantities). Finally, example 3 gives insight into the multi-modal process of meaning-making within the small group discussion: the students determined the relevant quantities (compare example 1), they focused, prepared and organized the (for them relevant) information (compare example 2), and they now interpret and give meaning to the phenomena using different (mathematical) strategies and corresponding mathematical gestures, as well as verbal and written signs. These insights are closely linked to the categories of Thompson and Carlson (2017) in such a way that their procedural character provides insight into the emergence, evolution and connection between the different covariation levels.

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